Achieving Teacher Professional Growth Through a Focus on Making Students' Mathematical Thinking Visible

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Providing opportunities for students to demonstrate their mathematical thinking can be challenging. This paper reports on a case study conducted with a teacher and her class of Grade 3/4 students which investigated this phenomenon. Following collection of baseline data that showed her students were not demonstrating evidence of mathematical thinking, the teacher implemented teaching approaches designed to address this. The findings indicated the teaching approaches were effective and led to changes in the teacher's practice and professional growth. The study has implications for teachers who are looking to make changes to their practice based on evidence-informed teaching approaches.

The importance of students learning mathematics with conceptual understanding and being provided with opportunities to demonstrate their thinking is promoted in the research literature and curriculum documents (e.g., ACARA, 2022; National Research Council, 2001). While research has provided us with a shared understanding of what constitutes quality mathematics teaching (e.g., Sullivan, 2011), there is wide variability in how effective teaching approaches are implemented in mathematics classrooms (Callingham, et al., 2017). Evidence suggests that mathematics lessons that are dominated by teacher-led activities and practices do not facilitate student thinking, or deep learning (Hattie, et al., 2017).

Research has shown that effective teaching approaches for mathematics include provision of appropriately challenging tasks, a focus on purposeful discussion, and an expectation that students can explain, reason, and justify their thinking (e.g., Sullivan, 2011). However, even with the provision of suitably challenging tasks and appropriate teaching approaches as advocated by Sullivan (2018), there is no guarantee that students will engage in problem solving and thinking behaviours. What is required is a shift from traditional practices and institutional norms which can be addressed through teacher Professional Learning (PL). If teachers are to make changes to the ways in which they teach mathematics, they require opportunities to be exposed to alternative practices, together with support to implement and reflect on the effectiveness of the practices, and their impact on students' learning (e.g., Muir & Beswick, 2007). Researchers have also identified that changes in teachers' practice are more likely to occur if the PL occurs in teachers' immediate environments (Loucks-Horsley, 2003), involves the use of a knowledgeable other (Timperley, 2008), and results in salient outcomes (Clarke & Hollingsworth, 2002).

This paper reports on a case study conducted with a teacher who engaged in collaborative practitioner research, with a view to improving her practice. The case study was situated in a real educational context, and focused on the trial and evaluation of Liljedahl's (2021) teaching practices for enhancing learning. The study complements and builds on Liljedahl's (2021) research in that it provides an in-depth account of how one teacher worked collaboratively with a researcher to implement three specific practices in a multi-year class in Australia. Liljedahl's reported findings offer accounts of examples of implementation from 40 different classrooms in Canada, rather than in-depth accounts of the benefits and challenges experienced by individual teachers as they implemented the strategies over a two-week period. Specifically the research questions were:

- What teaching approaches facilitate mathematical thinking and make it visible?
- What changes in a teacher's professional growth can occur as an outcome of implementing these approaches?

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Background Literature

Studenting Behaviours

The term 'studenting', first coined by Fenstermacher (1986), describes the behaviours that students do to help themselves learn, including paying attention to following instructions, seeking help, and studying. He later expanded the term to include other behaviours that students do that do not help them to learn, such as avoidance behaviours. Liljedahl (2021), who has conducted extensive research in this area, identified five common studenting behaviours, that were manifested when students undertook 'now you try one' activities:

Slacking. No attempt at task; talking; doing nothing.

Stalling. No real attempt at task; legitimate off-task behaviours such as pencil sharpening.

Faking. Pretending to do task, but achieving nothing.

Mimicking. Essentially copying the teacher, others or worked examples; attempt and often complete task.

Trying on own. Use reasoning and understanding to work through task.

Liljedahl's research found that the majority of students' time was spent on 'mimicking' (53%) and that there was little evidence of actual thinking behaviour observed across the 40 classrooms in which his research was conducted.

Practices that Generate Thinking

In order to address the prevalence of studenting non-thinking behaviours, Liljedahl (2021) identified 14 teaching practices that facilitated student thinking. These included types of tasks, where students work, forming collaborative groups, and when, where, and how tasks are given (for more see Liljedahl, 2021). The 14 practices provided a means for teachers to replace their familiar patterns of teaching, with different, more effective, field-tested techniques. As a result, changes occurred in teachers' practice not for ideological reasons, but because implementation of the practices led to increased student thinking in the classrooms studied (Liljedahl, 2021).

The types of mathematical tasks that teachers choose to use in their classrooms determines what content students learn, and also how they think about, use and make sense of mathematics (Stein, et al. 1996). Tasks should not focus on right answers, but instead provide opportunities for students to struggle with ideas and to develop a range of mathematical processes (Anthony & Walshaw, 2009). Research-informed practices (e.g., Sullivan, 2011) include the use of challenging tasks that go beyond a focus on procedural fluency, and require students to select their own strategies, develop persistence, and explain their thinking. When teaching using challenging tasks, teachers are encouraged to provide minimal guidance to students when introducing the task, and affirm positive behaviours such as persistence, effort, cooperation, and flexible thinking. (Sullivan, 2018).

Theoretical Framework

Professional Growth Model

Clarke & Hollingsworth's (2002) model of professional growth (see Figure 1) illustrates how an external source of information or stimulus provides an opportunity for teachers to engage in professional experimentation related to their practice. Consistent with Guskey's (1986) seminal change work, the model supports the premise that changes in teachers' beliefs and attitudes are primarily derived from classroom experiences and likely to occur following evidence of improved student outcomes. As Figure 1 depicts, the model contains four domains, with each domain connected through the mediating processes of enactment and reflection. Professional growth occurs when changes in the personal domain (knowledge, beliefs and attitudes) occur as a result of external

stimulus, leading to professional experimentation (such as implementing new teaching strategies) and experiencing salient student outcomes (such as improvements in students' mathematical dispositions). The model was used as a framework to guide the collaborative inquiry process. The researcher acted as an external source of information or stimulus, and the teacher engaged in professional experimentation as she trialled different approaches over the course of a term. Reflective conversations occurred around noting the impact of the approaches on student outcomes, which were often salient and both the researcher and teacher noted changes in knowledge, beliefs and attitudes.

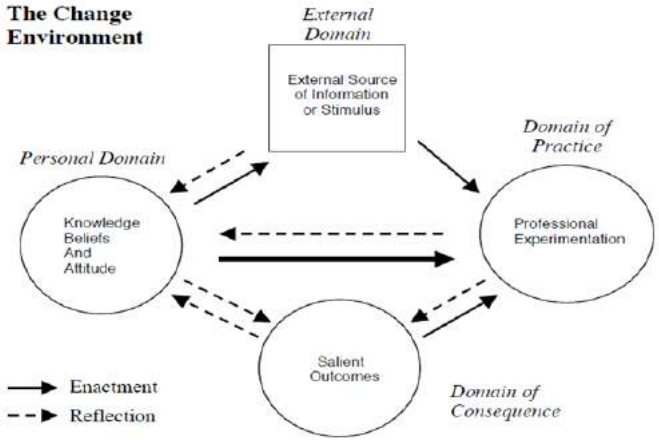

Figure 1. Model of professional growth (Clarke & Hollingsworth, 2002).

Methodology

The study used a case study paradigm to explain and describe an event or phenomena in the everyday context in which they occurred (Yin, 2009). In this instance, the case study is used to understand and explain the impact of three different teaching approaches undertaken by a teacher involving her class of Grade 3/4 students. The adoption of specific teaching approaches constitutes the bounds of the case (Stake, 1995).

Context

The study occurred in an Australian urban primary school with a Grade 3/4 teacher and her class of 25 students. The teacher 'Shandi' (pseudonym) had eight years of teaching experience, four years at that school and four years of teaching that grade level. She was motivated to participate in the study because she wanted to improve student engagement, and her experience of mathematics PL was limited to the occasional conference and in-school sessions. Full ethical approval was obtained, and consent provided for all participants.

Procedure and Data Collection

The study took place in Term 4 in 2021, over the course of nine weeks. Initial base-line data were collected in two lesson observations to record evidence of studenting behaviours using a time-scale checklist. Unlike Liljedahl's research, the lesson observations were not restricted to 'now you try one' experiences. Entries were made at 5 minute intervals, with a different student observed at each interval. A total of 11 lesson observations occurred throughout the term, with each observation including a completed checklist and field notes. Teacher interviews were conducted following each lesson observation whereby professional conversation and teacher reflections on the lesson occurred, along with identification of when and how to implement future targeted teaching strategies. Following the initial two-week observation period, the iterative cycle involved the selection of an intervention (teaching strategy), implementation of the strategy over two weeks (approximately 5

lessons) and evaluation and reflection on the impact of the strategy. This cycle was repeated three times, with a total of 11teacher interviews being conducted.

Data Analysis

Data from the teacher interviews were analysed using both an inductive and deductive approach (Braun & Clarke, 2022). Firstly, the interviews were fully transcribed and entered into NVivo. Open coding was initially applied, and instances of references to studenting behaviours and evidence of aspects of application of the four elements in the Model of Professional Growth were deductively noted. It was therefore possible for multiple codes to be applied to sections of the data. For example, the following excerpt was coded to salient outcomes; student behaviour—trying; challenging tasks:

> I'm pretty impressed with what I got out them today. So not necessarily if they got the answer right or wrong—just getting things down. Having a go. Being challenged and being okay with being challenged.

Results and Discussion

In this section the findings from the teacher conversations are presented and discussed. The findings are structured according to the four elements of the model, beginning with detailing the external stimulus provided by the researcher and the student checklist results, and culminating in evidence of shifts in the teacher's beliefs and attitudes. The section concludes with a brief discussion that addresses the research questions.

External Source of Information or Stimulus

The researcher was the primary external source of information which included familiarising Shandi with Lildjedahl's (2021) research into studenting behaviours and recommended teaching practices, and Sullivan's (2018) challenging task approach. Base-line data collected from two initial lesson observations were shared with her and professional conversation took place about the presence and regularity of non-thinking student behaviours. Checklist data showed that 63% of student behaviours observed were non-thinking behaviours, and 26% 'trying' behaviours. Interestingly, another category was generated as 11% of behaviours were best described as 'watching' as any mathematical thinking was not visible. The most prevalent studenting behaviours were slacking and stalling (54%) and included rubbing out, sharpening pencils, and playing with counters. These behaviours were especially dominant in unsupervised group work. In summary, lesson observations indicated there was evidence of high levels of non-thinking student behaviour occurring in mathematics lessons. The next step was then to identify which approaches to trial to address this issue.

Professional Experimentation and Salient Outcomes

Professional experimentation is situated within the Domain of Practice and involves the trialling of an intervention or strategy. Shandi initially selected the provision of challenging tasks to trial as she felt a lot of her practice was text-book based and dominated by a focus on rules and procedures. Subsequent teaching strategies included random groupings and use of vertical writing surfaces (how, when and where tasks are given). Professional conversations tended to link the strategies with impact upon students' behaviour, hence this section is structured around Shandi's reflections on each teaching strategy trialled.

Challenging Tasks

Shandi was "a little familiar with challenging tasks—I've had a little time working with them but not to a great extent" [initial tr interview]. Following a lesson where students were asked to individually record how many different picture frames they could depict that had a perimeter of 24 cm, Shandi explained her approach and what she noticed about the students' thinking behaviour:

> I chose the middle question from the open-ended questions book. I didn't give them any hints—I love that! So they do know this without us telling them so it's getting them thinking.

Shandi continued to source most of her challenging tasks from Sullivan's teacher resource books (Sullivan, 2018; Sullivan & Lilburn, 2006), and then complemented them with Liljedahl's teaching strategies. Towards the end of the term, Shandi shared that in the past she avoided giving students open-ended tasks as:

> They'd be under desks or leaving the room or having full meltdowns, because they would be like, no can't do it. Don't know what the question is—can't do it. Whereas now they don't even ask. They know it's open ended. And I think that's a big thing as well—setting them up for that. And they're definitely more willing to have a go.

Random Groupings

Prior to participating in the study, Shandi's approach to group work was to carefully select group members based on their personality and ability level. She would typically have a teaching group of four or five students of similar ability, with the remainder of the class allocated to small groups who had different rotating activities to complete. As noted earlier, studenting non-thinking behaviours were especially prevalent in groups that were working independently from the teacher on set activities. In her first post-lesson conversation Shandi expressed her preference for similar ability grouping:

> I like the fact that I can work with each group based on ability. I know that ability grouping sometimes is not always the best, and I've tried to do some different strategies for that, but I think, for that explicit teaching, I do find it beneficial having the ability groups for that.

In the same conversation she noted that:

> Group work is not this group's strong point—not at all. They're OK with pairs, but … lots of tantrums when it comes to group work.

Shandi was initially hesitant about the concept of random grouping, mainly due to behavioural management considerations. When trialling this strategy, she used playing cards or random name generators to create groups of three. She provided the following observations regarding her professional experimentation and outcomes for students as they experienced random groupings:

> I think yesterday when we first did the groups, they found it a little bit challenging working with each other. But then today, it was a different kettle of fish. They were fantastic.

> I like the fact that they were random, and it gave kids who don't normally work together, or wouldn't even remotely think of joining a group together, the chance to do so. And Jack actually said that he didn't know too much about the two people he worked with and so he had to listen to what they were saying because their thinking was different to his—so I'm thinking, wow!

> I think, last week, three times in a row, I had three boys together, just at random, it wasn't anyone's doing. And they actually worked really well together. Whereas normally I would be 'No—they're not working together'. So yeah, that's good—but it has been hard to let that go.

Use of Vertical Writing Surfaces

Once the random groups of three were created, the students were given a challenging task to collaboratively solve and record their thinking on vertical writing surfaces. As the classroom was not equipped with vertical writing surfaces, other than a small whiteboard, Shandi improvised and used A3 sheets of paper taped to walls and windows for the groups to record on. Students worked collaboratively on tasks, with the restriction that the student recording the answers could only record others' thinking, not their own. Shandi was particularly enthusiastic about the impact of this strategy:

> I think it's fantastic. I thought it was really good. It gets them all standing. So they don't have the ability to slack off as much, I think, because they all have a turn of the pen. The people that don't have the pen have to do the thinking. So it's a real group effort. Everyone has something to do, which I think in normal groupings, or activities that I've done, you have the people that will do the slacking or the avoiding, and not engaging

> with what's happening, with what they're doing. But this sort of takes that away. So yeah, I think that has worked really well.
>
> I think them standing and standing up looking at the process, as they do in the vertical way keeps them super engaged, because they don't have a … chance to go and wander around because they have to be with their group of three. Yes, there's a lot more responsibility on them to participate.

Shandi also felt that the combined use of small groupings and the vertical surfaces had a positive impact upon making students' thinking visible:

> I think because they have to articulate to each other their thinking instead of just the one person scribing, they have to actually be able to talk about the maths—because there's no way around it. And then having them change the pen—they're all accountable. So I think that's really improved that discussion, and the articulation of what's actually happening with their mathematical thinking.

For Shandi, the most saliant outcome was students' increased resilience. In her last post lesson interview she stated that:

> [The biggest impact on students] has been their resilience. So their resilience to actually work through 'I don't know how to do this' to think' right—how else can I do this?' I think that's been massive, huge.

Knowledge, Beliefs and Attitudes

In her first interview, Shandi revealed that when she reflected on her practice ("what I'm doing, and how I'm doing it"), she believed, "this is definitely not working" and asked herself, "What can I do? How am I going to change? How am I going to help them [students] out?" In the same interview, she self-reflected that:

> When I'm doing that explicit teaching I think sometimes I lose a few, so less me talking and more them doing, which is something I've always had trouble with, and letting go a little bit more. I think that in writing it's all 'I do, we do, you do' whereas in maths I need them to be doing it from the get go.

Shandi's comments indicated that there were aspects of her practice that she wanted to improve and was therefore receptive to trialling new approaches. Evidence of a shift in her knowledge, beliefs, and attitudes was apparent in ongoing post-lesson interviews when she reflected positively upon the students' engagement in the lessons, students' improved resilience, and the increase in thinking and on task behaviours. Furthermore, in her final interview, she stated that the biggest impact on her was:

> Letting go a bit more and not having such a tight hold on everything. I've had to go outside my comfort zone and be OK with the groups not being perfect. I've learned to be OK with them talking and being a bit louder whereas I think for me before, they were nice and quiet and relaxed, whereas this is a little bit more hectic. But for me, I've got to let that go, and it's OK because I know they're on task.

Participation in collaborative inquiry not only led to a shift in Shandi's beliefs, it also broadened her knowledge of teaching strategies. In summary, Shandi trialled the use of challenging tasks, and associated pedagogical approach, random groupings, and use of vertical writing surfaces. In her final interview, she confirmed that she would continue to implement the strategies and "that they could all be incorporated in some way, shape or form throughout all units of maths and in other areas as well". She questioned her previous approaches, such as "doing all that whiteboard stuff on the floor and them just copying what I'm doing" and the use of worksheets as "not showing that thinking—it's just copying or following the formula". Shandi identified that there would still be a place for "explicit teaching because sometimes you want to be able to copy to get it in some situations—it's a tricky one—but I think I'm relying on that too much".

Teaching Approaches That Facilitate Mathematical Thinking

The 14 practices that were identified by Liljedahl (2021) as generating more thinking than institutional normative practices were subjected to trials and iterations until they became 'optimal practices for thinking' (p. 16). Like the teachers in Liljedahl's (2021) research, Shandi trialled each

approach for two weeks, but in contrast to his teachers, she engaged in regular post lesson interviews with the researcher. The findings from Shandi's case study indicate that the three practices that she trialled with her class did facilitate mathematical thinking, and also developed students' dispositions to attempt tasks, persist and demonstrate resilience, likely influenced by the implementing challenging tasks approach (Sullivan, 2018).

The use of challenging tasks reduced the tendency for the students to copy or mimic the teacher—a dominant non-thinking studenting behaviour in both Liljedahl's research and consistently referred to by Shandi. While mimicking may be useful for teaching students how to replicate routines, it tends to happen not alongside of, but instead of, thinking (Liljedahl, 2021).

Visible random groupings facilitated thinking behaviour in that the mismatch that often occurs between the teacher and the students' goals when groups are strategically or self-selected, was removed (Liljedahl, 2021). Instead students did not enter into group work 'feeling like they were going to be a follower rather than a leader (Liljedahl, 2021, p. 41), provided the students believed that the groups were genuinely random. Consistent with Liljedahl's findings, groups of three were the optimal group size.

Liljedahl's (2021) research found that students standing and working on vertical surfaces had several advantages, including making their work visible, removing students' sense of anonymity, and increased accountability. In Shandi's classroom, this practice was optimised when students not only took turns having the pen, but were only allowed to record others' ideas. This seemed to result in an even distribution of contributions and an expectation that group members worked collaboratively.

While student mathematical outcomes were not evaluated, classroom observations reinforced Shandi's perceptions that students were on task, with less evidence of non-thinking behaviours than earlier in the term. This observation was reinforced from the final lesson observation where there were no instances of mimicking or stalling behaviour; instances of slacking behaviour (24%) were noted however. As indicated earlier, an additional category was added to the sudenting behaviours to accurately depict that 'watching' was commonly observed, which was not easily attributable to slacking, stalling or faking behaviour.

Teacher Professional Growth

Clarke & Hollingsworth's (2002) model proved useful in interpreting Shandi's professional growth. Evidence from Shandi's interviews demonstrate that she was open to professional experimentation of teaching strategies provided through an external stimulus. Furthermore, she noted improvements in students' dispositions to undertake tasks and to work collaboratively, and a decline in non-thinking behaviours. These salient outcomes likely influenced her commitment to continue with trialling subsequent approaches, reflecting on their effectiveness, and identifying future teaching approaches. Consistent with effective professional learning principles, teacher change is more likely to occur and be sustained if it is situated within realistic contexts and teachers are given multiple opportunities to trial new approaches and evaluate the impact of these approaches (Timperley, 2008). Shandi's engagement with, and commitment to the process, including opportunity to engage in post-lesson reflections with the researcher, created the optimum conditions for professional growth to occur.

Conclusions and Implications

The case study presented in this paper provides evidence to support Liljedahl's (2021) assertion that non-thinking studenting behaviours are prevalent in today's mathematics classrooms. It is possible, however, to reduce the prevalence of these behaviours through the use of teaching practices that enhance and promote student thinking. Through being exposed to these practices through the input of an external stimulus (the researcher), and supported to engage in professional

experimentation through trialling selected practices, Shandi demonstrated professional growth in her teaching of mathematics. Like Ms Duo (Liljedahl & Allan, 2013), Shandi changed aspects of her practice, with her involvement in the collaborative inquiry process a powerful catalyst for initiating teacher change. She was motivated to sustain these changes as she observed and reflected upon the positive impact of the trialled teaching strategies on her students. In this sense, Shandi experienced a form of very individualised, supportive professional learning that occurred in the realistic context of her classroom.

This case study has implications for school leaders and policy makers in terms of valuing what constitutes effective professional learning and resourcing it accordingly. While involving teachers in collaborative classroom research may be time consuming and resource intensive, the benefits are likely to be worth the investment. For teachers, the practical implications are that reflecting on practice with an external colleague or educator, along with observing salient student outcomes, provides the impetus to continue to engage in professional experimentation. Implementing some of Liljedahl's (2021) 14 teaching strategies could form the basis of that experimentation given that this case study has added to the research evidence that the strategies are effective in increasing students' thinking behaviours. Future studies could examine the impact and effectiveness of other strategies, along with trialling how collaborative research into practice can be upscaled to develop teacher capacity and engage teachers in authentic individualised professional learning experiences.

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